

# Midterm Exam Calculus 1

23 september 2013, 9.00-11.00.

Write on each page your name and student number, and on the first page your seminar group. The use of annotations, books and calculators is not permitted in this examination. All answers must be supported by arguments/work. Success.

1. (a) Formulate the principle of mathematical induction.  
(b) Prove that if  $n \geq 1$  is a positive integer, then

$$2^1 + 2^2 + \dots + 2^n = 2^{n+1} - 2$$

2. Prove that for all integers  $n \geq 1$ ,

$$8^n - 3^n$$

is divisible by 5.

3. Solve

$$z^2 = \frac{1}{1-i}$$

and sketch the solutions in the complex plane.

4. Determine all complex numbers  $z$  satisfying

$$e^{iz} = -1$$

Maximum score:

1a 1.0 2 2.0 3 2.0 4 2.0  
b 2.0

Total: 9 +1 (free) = 10.